FINITE DIFFERENCE AND MULTIPLE REGRESSION METHODS APPLIED IN DYNAMIC MODELING OF WIND ENERGY

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ABSTRACT

The intermittency of wind energy poses challenges to the stability of the electrical grid, making the precise modeling of the relationship between wind speed and generated power crucial. Traditional models, based on static algebraic equations, are insufficient as they fail to capture the dynamics of the wind and the non-instantaneous responses of the turbine, such as inertia and elasticity. This paper aims to overcome these limitations by developing a dynamic model for wind power, based on a differential equation that describes the temporal evolution of the system. The methodology integrates physical modeling with rigorous statistical analysis, using the Finite Difference Method for the numerical solution of the differential equation, based on classic works in the field. For the calibration and validation of the model against empirical data, the Multiple Regression Method was employed, which allows for analyzing the relationship between power and multiple predictors and evaluating the residuals of the dynamic model. As a result, a particular differential equation describing the dynamic behavior of power was obtained, demonstrating superiority over the static approach, especially under transient wind conditions. The proposed hybrid approach not only offers a more accurate solution for generation forecasting but also establishes a foundation for optimizing control algorithms and future research in the field.

Keywords: Wind Turbines; Differential Equations; Numerical Analysis; Power Forecasting; Computational Simulation.

MÉTODOS DE DIFERENÇAS FINITAS E REGRESSÃO MÚLTIPLA APLICADOS NA MODELAGEM DINÂMICA DA ENERGIA EÓLICA

RESUMO

A intermitência da energia eólica impõe desafios à estabilidade da rede elétrica, tornando crucial a modelagem precisa da relação entre a velocidade do vento e a potência gerada. Modelos tradicionais, baseados em equações algébricas estáticas, são insuficientes por não capturarem a dinâmica do vento e as respostas não instantâneas da turbina, como inércia e elasticidade. Este trabalho propõe superar tais limitações desenvolvendo um modelo dinâmico para a potência eólica, fundamentado em uma equação diferencial que descreve a evolução temporal do sistema. A metodologia integra a modelagem física com a análise estatística rigorosa, utilizando o Método de Diferenças Finitas para a resolução numérica da equação diferencial, com base em trabalhos clássicos da área. Para a calibração e validação do modelo frente a dados empíricos, foi empregado o Método de Regressão Múltipla, que permite analisar a relação da potência com múltiplos preditores e avaliar os resíduos do modelo dinâmico. Como resultado, foi obtida uma equação diferencial particular que descreve o comportamento dinâmico da potência, demonstrando superioridade sobre a abordagem estática, especialmente em condições de vento transientes. A abordagem híbrida proposta não só oferece uma solução mais precisa para a previsão de geração, mas também estabelece uma base para a otimização de algoritmos de controle e futuras investigações na área.

Palavras-chave: Aerogeradores; Equações Diferenciais; Análise Numérica; Previsão de Potência; Simulação Computacional.
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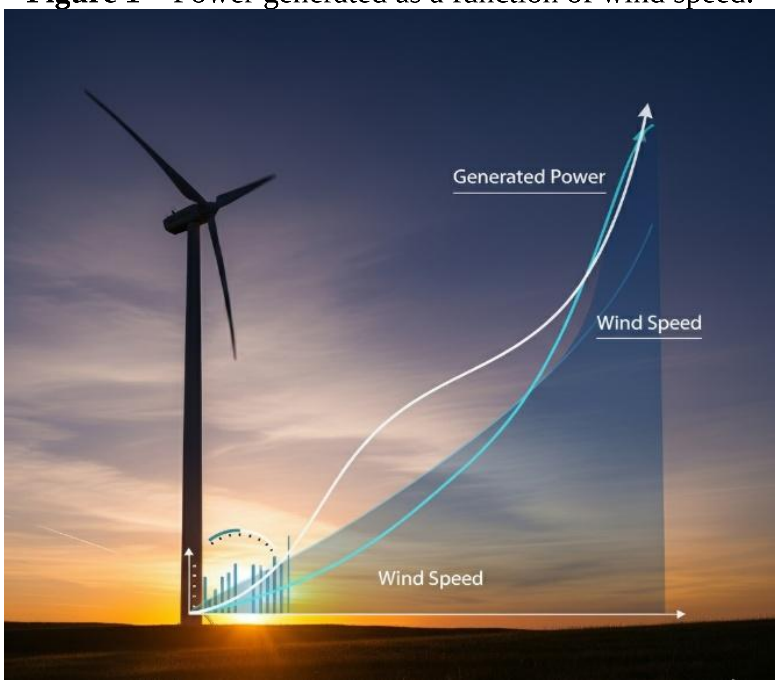
INTRODUCTION

Wind power generation represents one of the cornerstones of the transition to a sustainable energy matrix. However, its intermittent nature imposes significant challenges to the predictability and stability of the electrical grid. The precise modeling of the relationship between wind speed and the electrical power generated is, therefore, a task of fundamental importance. Traditionally, this relationship, highlighted in Figure 1, is described by a static algebraic equation, which, although useful, fails to capture the dynamic nature of both the wind and the turbine itself, which possesses inertia, elasticity, and non-instantaneous electromechanical responses.

Figure 1 – Power generated as a function of wind speed.



Source: Authors, 2026.

To overcome these limitations, the modeling can be approached through differential equations that describe the temporal evolution of the system, treating the rotational speed as a state variable governed by a balance of torques. The complexity of such equations, especially in nonlinear systems, often precludes an analytical solution, thus necessitating the use of robust numerical methods (MA et al., 2019). Consequently, the Finite Difference Method emerges as a classic and powerful approach for the numerical resolution of these problems. The theoretical foundations and the analysis of stability and convergence for such methods were rigorously established in seminal works like those by Forsythe and Wasow (1960) and Keller (1992), which remain fundamental references in the field of numerical analysis.

Beyond the physical formulation and numerical solution, the construction of a reliable model requires a thorough analysis of operational data for calibration, validation, and identification of influential secondary variables. In this context, the Multiple Regression Method becomes an indispensable statistical tool. As detailed in comprehensive works such as Pedhazur (1997), Multiple Regression allows for the quantification of the relationship between the power generated and multiple predictors simultaneously (e.g., wind speed, air density, turbulence), as well as the validation of the differential model's results against empirical data. The practical application of this method, as demonstrated by Tavares et al. (2018) in a different context to identify significant factors in a complex system, illustrates its power in discerning the main control variables.

This article, therefore, aims to develop a dynamic model for wind power based on a differential equation, which will be solved numerically based on the principles of the Finite Difference Method. Additionally, the study will employ the tools of Multiple Regression to analyze the model's residuals and validate its accuracy against real data, thereby integrating physics-based modeling from first principles with rigorous statistical analysis.

MATERIALS AND METHODS

Wind power generation, one of the fastest-growing renewable energy sources in the world, is governed by a fundamental equation that relates the power available in the wind to factors such as air velocity, atmospheric density, and the characteristics of the wind turbine. Understanding this formula is essential to grasp the potential and limitations of wind energy. The primary representation of the power generated by a wind turbine is shown in Equation (1).

$$P = 0.5\rho\eta A C_p v^3 \qquad (1)$$

Where, $\boldsymbol{P}$ is the Power generated (W); $\boldsymbol{\rho}$ is the air density (kg/m³), which in turn varies with temperature, atmospheric pressure, and altitude. A common reference value at sea level is approximately 1.225 kg/m³; $\boldsymbol{\eta}$ is the generator efficiency; $\boldsymbol{A}$ is the rotor swept area (m²); $\boldsymbol{v}$ is the wind speed (m/s); $\boldsymbol{Cp}$ is the power coefficient, a dimensionless value representing how efficiently the wind turbine can convert the kinetic energy of the wind into mechanical energy.

It is important to note that a wind turbine cannot convert 100% of the wind's kinetic energy into electricity. There is a theoretical maximum limit for the power coefficient, known as Betz's Law (SILVA, J., 2023). According to this law, the theoretical maximum value of $\boldsymbol{Cp}$ is approximately 0.593 or 59.3%. This limit exists because, for the turbine to extract energy, the wind needs to pass through it and continue its flow at a lower speed. If the turbine extracted 100% of the energy, the wind would stop completely behind the blades, preventing the passage of more air and, consequently, interrupting energy generation. Air density is of similar importance. According to Oliveira et al. (2014), variations in air density directly impact the performance and power curve of small wind turbines, which are frequently installed in locations with atmospheric conditions different from standard conditions.

The modeling of the Differential Equation, which represents the dynamic behavior of wind power generation, is based on Table 1, whose values were defined from the power curves presented in Medeiros (2008), Campos et al. (2022), Freitas et al. (2020), and Santos (2023). During its construction, it was necessary to assume fixed values for the other variables in Equation (1): Rotor Diameter: 160 m (resulting in a radius of 80 m). Swept Area (A) ≈ 20,106 m². ρ = 1.225 kg/m³ (standard value at sea level). η = 0.97 (or 97%). Cp = 0.45 (45% efficiency, a realistic value for modern turbines). Rated Power: 7,000 kW or 7.0 MW (the maximum power the generator can safely produce).

Table 1 - Relationship between wind speed and generated power.

Wind Speed (m/s)	Power Generated (kW)	Observations
1	0	Below the cut-in speed
2	0	Below the cut-in speed
3	149.8	Cut-in Speed - Start of Generation
4	355.1	-
5	693.5	The power practically doubles from 4 to 5 m/s.
6	1198.50	-
7	1919.20	-
8	2896.80	-
9	4172.50	-
10	5787.00	-
11	7781.50	-
12	10000 (limited to 7000)	Reaches the theoretical rated power
13	7000	Rated Power Reached (limited by the generator)
14	7000	Operating at rated power
15	7000	Operating at rated power
16	7000	Operating at rated power
17	7000	Operating at rated power
18	7000	Operating at rated power
20	7000	Operating at rated power
22	7000	Operating at rated power
24	7000	Nearing the cut-out speed
25	0	Cut-out Speed - Turbine shuts down for safety
26	0	Shut down for safety

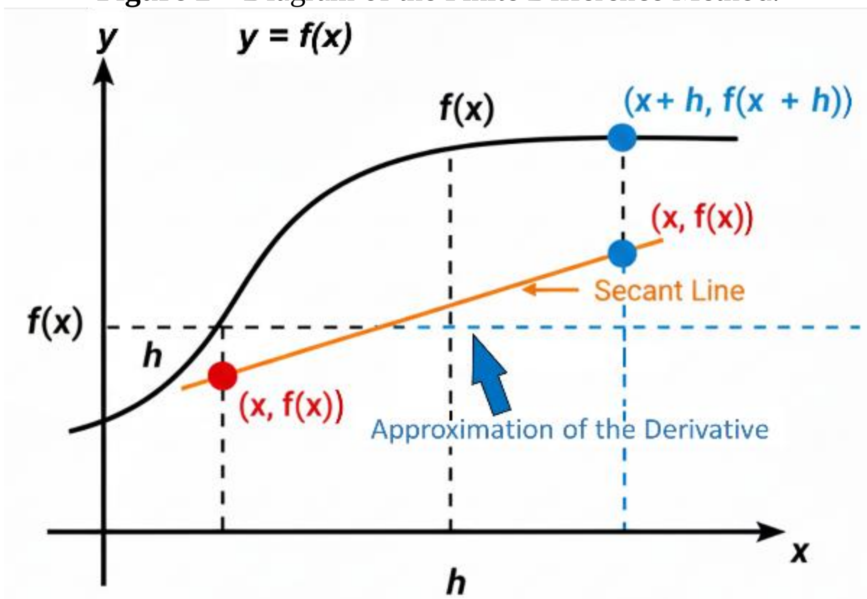
Source: Authors, 2026

Table 1 illustrates three important operational concepts of a real turbine: i) Cut-in Speed: The minimum wind speed required for the turbine to begin generating power (assumed here as 3 m/s). ii) Rated Speed: The speed at which the turbine reaches its maximum (rated) power. For speeds above this, the turbine adjusts its blades to maintain constant power and prevent damage to the generator. iii) Cut-out Speed: The maximum wind speed that the turbine can withstand. Above this value, it stops rotating for safety reasons (assumed here as 25 m/s). The power values are calculated until they reach the rated power of 7000 kW. From that point on, the power is electronically limited.

a) Finite Difference Method

In the construction of the Differential Equation, the Finite Difference Method (FDM) was initially applied, following Forsythe and Wasow (1960), Keller (1992), Zingg (2000), Forsyth and Vetzal (2017), and Tirtey (2018). The FDM is one of the essential and widely used numerical techniques for solving differential equations. The five cited scientific articles present details of the method, covering everything from its theoretical foundations to practical applications in various fields. Figure 2 shows a graphical representation of the Finite Difference Method.

Figure 2 – Diagram of the Finite Difference Method.

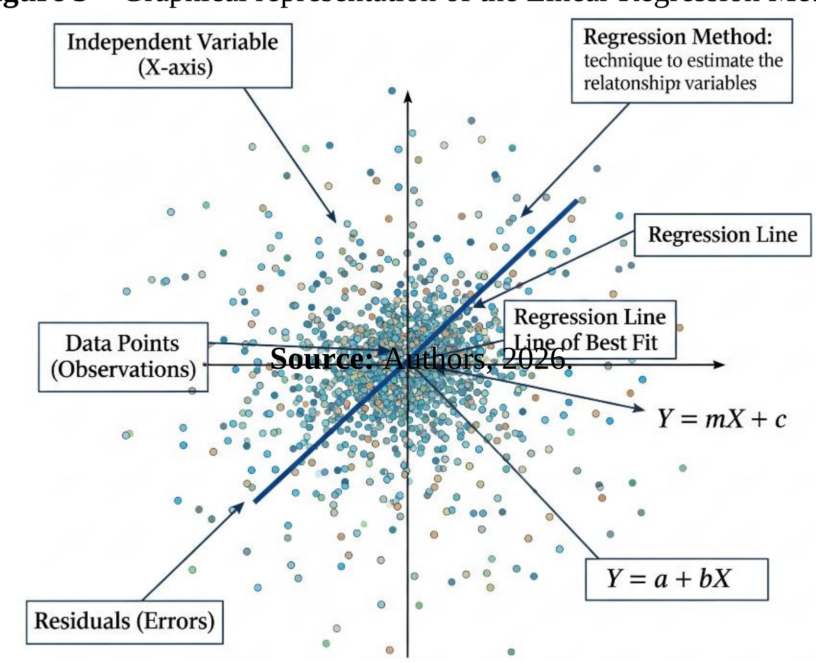


Source: Authors, 2026.

b) Multiple Regression Method

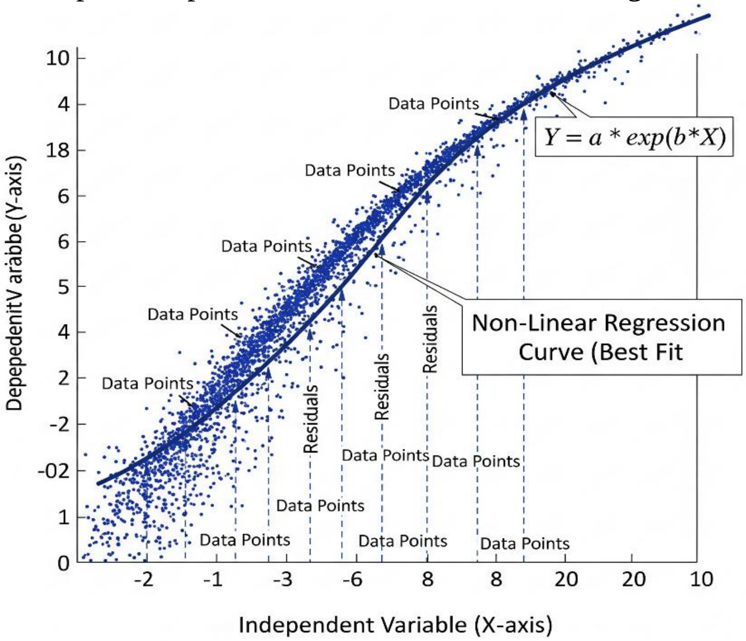
Based on the works of Bock (1963), Pedhazur (1997) and Tavares (2018), the Regression Method is, in its essence, a process for modeling the relationship between a dependent variable (also called the response or target variable) and one or more independent, or predictor, variables. Figures 3 and 4 show, respectively, two cases of regression: Linear and Non-Linear.

Figure 3 – Graphical representation of the Linear Regression Method.



Source: Authors, 2026.

Figure 4 – Graphical representation of the Non-Linear Regression Method.



Source: Authors, 2026.

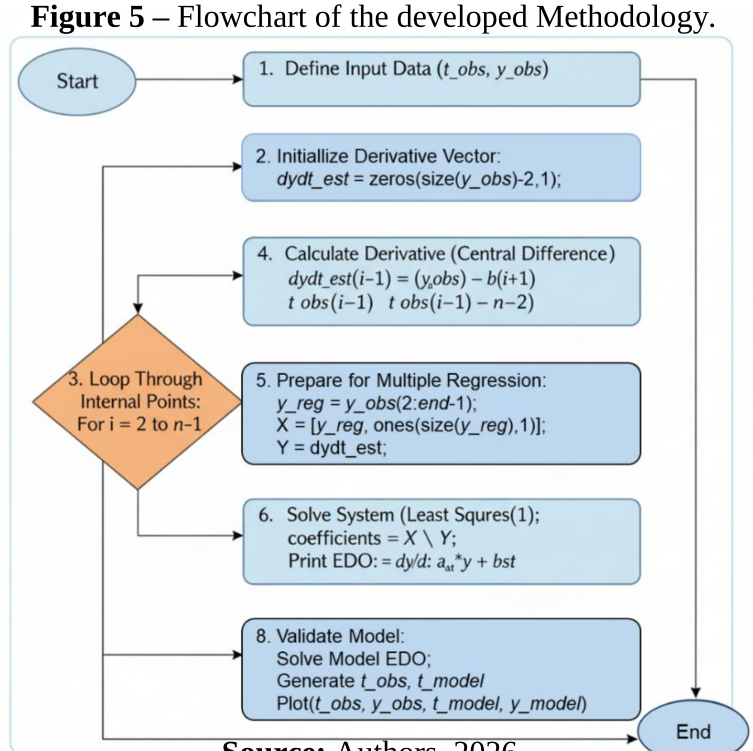
The objectives of Regression are:

i) To understand how the independent variables influence or relate to the dependent variable;

ii) To estimate the value of the dependent variable based on the values of the independent variables.

Figure 5 shows the algorithm for modeling the Differential Equation, summarizing the strategy applied in implementing this research.

Figure 5 – Flowchart of the developed Methodology.



Source: Authors, 2026.

RESULTS

The Mathematical-Computational Modeling of Power (P) as a function of velocity (v) was implemented using the OCTAVE scientific computing language (GNU, 2025). The structure of the Differential Equation is based on its general form, shown in Equation (2). For the phenomenon under study, the Differential Equation is represented by Equation (3).

$$\frac{dy}{dx} = f(x, y) \tag{2}$$

$$\frac{dP}{dv} = 0.2715P(v) + 233.4263 \tag{3}$$

The general solution of Equation (3) is defined in Equation (4).

$$P(v) = \varphi\, e^{0.2715v} - 515.8592 \tag{4}$$

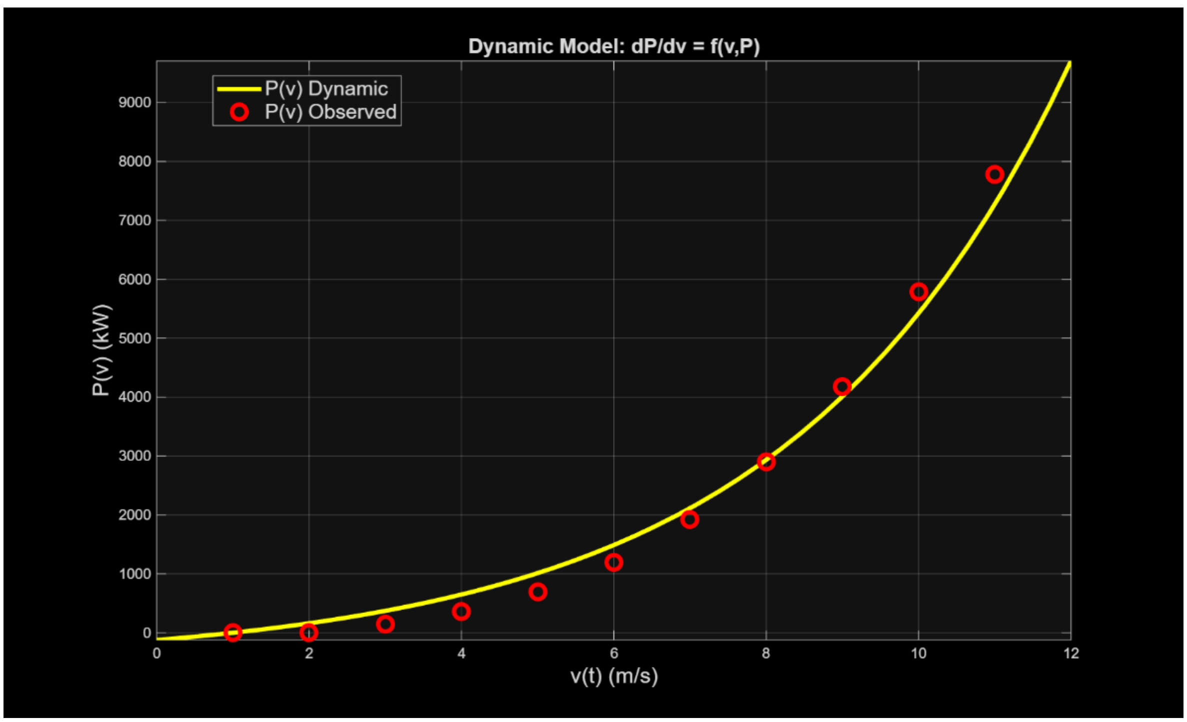
Where φ is an arbitrary constant.

Considering the initial and operational conditions, defined in Table 1, we have the particular solution of Equation (3) defined by Equation (5).

$$P(v) = (0.7622\, e^{0.2715v} - 1.3119) * 515.8592 \tag{5}$$

Figure 6 displays the dynamic behavior of the Power generated by a wind turbine as a function of wind Speed. The graph refers to the particular solution of Differential Equation (3), presented in (5).

Figure 6 – Representation of the Power of a Wind Turbine.



Source: Authors, 2026.

DISCUSSION

Representing the behavior of electrical energy in a wind turbine using a differential equation offers a crucial advantage over an algebraic equation: the ability to analyze and predict the dynamic and transient behavior of the system. While algebraic equations provide an instantaneous and static image of energy, differential equations reveal the temporal evolution and reactions of the wind turbine to variations, a fundamental aspect for the stability and control of modern electrical grids, unlike the proposal presented by Oliveira et al. (2018), which uses a statistical treatment of the uncertainties inherent in wind speed and measured power.

The main difference lies in the fundamental nature of each type of equation. An algebraic equation, in the context of a wind turbine, typically describes the electrical power as a direct function of wind speed and other parameters considered constant at a given moment. A simplified example would be Equation (1): This equation is extremely useful for calculating energy production in a steady state, that is, when conditions are stable. However, it fails to capture how the wind turbine behaves during rapid fluctuations in wind speed, during faults in the electrical grid, or throughout the turbine's start-up and shutdown processes.

It is at this point that differential equations become indispensable. They model the rates of change of the system's quantities, incorporating the turbine's inertia, the generator's electromechanical characteristics, and the response of the control systems. Electrical power is no longer seen as an instantaneous value but as a variable that evolves over time, influenced by the previous state of the system. The differential equations that govern the dynamics of a wind turbine take into account, for example:

i) The equation of motion for the rotor: which relates the angular acceleration of the turbine to the difference between the mechanical torque extracted from the wind and the electromagnetic torque of the generator.
ii) The electric generator equations: which describe the dynamics of currents and voltages in the stator and rotor windings, fundamental for stability and short-circuit analysis.
iii) The power converter equations: which model the response of the electronic components that interface between the generator and the electrical grid.

Table 2 shows the main advantages of modeling with differential equations, providing a comparative analysis between algebraic (static) and differential (dynamic) equations, with a specific focus on their application in grid-connected wind turbine systems. While algebraic equations offer a simplified view of steady-state conditions, differential equations are indispensable for the challenges of modern engineering involving stability, control, and grid integration.

Table 2 – Comparison between the Differential and Algebraic models for the behavior of generated Power.

Characteristic	Algebraic Equation (Static)	Differential Equation (Dynamic)	Advantage of the Differential Equation
Temporal Analysis	Provides an instantaneous snapshot of power under stable conditions.	Describes the evolution of power over time.	Allows for predicting how the system will react to disturbances and changes.
Stability	Cannot predict instabilities.	Essential for studies of transient and small-signal stability.	Ensures the safe and reliable operation of the grid-connected wind turbine.
Control	Inadequate for designing dynamic control systems.	Basis for the development of real-time control loops.	Optimizes energy extraction and ensures the quality of the power injected into the grid.
Transients	Ignores transient phenomena.	Captures the behavior during events such as wind gusts and grid faults.	Allows for the design of adequate protections and for understanding the wind turbine's impact on the grid.

Realism	Simplified representation of reality.	A more faithful model of the system's actual physical behavior.	Increases the accuracy of simulations and the reliability of analyses.

Source: Authors, 2026.

While the algebraic equation is a tool for the macro-level evaluation of energy production, the differential equation is the tool that allows for a profound understanding essential for robust control systems and for ensuring the safe and efficient integration of wind turbines into complex and dynamic electric power systems. The increasing use of renewable energy sources, such as wind power, makes this dynamic analysis increasingly critical for the stability and resilience of the electrical grid.

CONCLUSIONS

This work has proposed and detailed a robust methodological approach for modeling the power generated by wind turbines, overcoming the intrinsic limitations of traditional static models. The main contribution of this research lies in the synergistic integration of a physical-dynamic model, based on a differential equation, with well-established numerical methods and rigorous statistical validation, creating a complete and high-fidelity framework for the analysis of wind power performance.

The presented methodology cohesively articulates the formulation of the rotor's dynamics via a differential equation that captures the system's inertia, with its practical solution made feasible by the Finite Difference Method. Grounding this stage in classic works of numerical analysis, such as those by Forsythe and Wasow (1960) and Keller (1992), ensures rigor in the transposition of the theoretical model into a reliable computational simulation. Additionally, the incorporation of the Multiple Regression Method, following the guidelines of Pedhazur (1997), plays a dual and crucial role: first, it establishes a statistical benchmark for performance comparison; second, and more importantly, it acts as a diagnostic tool to analyze the residuals of the dynamic model, allowing for the identification of error sources and the influence of variables not contemplated in the initial physical formulation, in an approach analogous to that of Tavares et al. (2018).

It is expected that the application of this methodology will result in a wind power model with superior predictive accuracy, especially under transient and turbulent wind conditions where static models prove inadequate. The ability to simulate the dynamic response of the wind turbine has direct practical implications for the engineering and operation of power systems: it enables the optimization of pitch control algorithms, improves short-term generation forecasting for grid management, and allows for a more precise assessment of energy yield in locations with complex wind profiles.

The contributions of this study also open avenues for future research. The model's structure allows for the incorporation of more detailed physical effects, such as blade flexibility or aerodynamic interaction in the turbine wake. Furthermore, the numerical solution can be enhanced with higher-order schemes, such as those explored by Zingg (2000), to increase accuracy in long-term simulations. It is also recommended to use Machine Learning and Deep Learning models in modeling the power curve of wind turbines, using real data from wind farms, as presented by Medeiros et al. (2020). In summary, the hybrid approach detailed here not only provides a more accurate solution to a classic problem in renewable energy engineering, but also establishes a solid and expandable foundation for future investigations in the field.

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